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Let  $m=0$ .

$$\begin{aligned}\therefore V &= \frac{1}{3}n^2c^3 \left[ \frac{1}{2}\pi - \sin^{-1} \left( \frac{a}{nc} \right) \right] \\ &+ \frac{a^3}{3n} \log \left( \frac{nc + \sqrt{[n^2c^2 - a^2]}}{a} \right) - \frac{2}{3}ac\sqrt{[n^2c^2 - a^2]} \\ &= \frac{1}{3}c \left[ \frac{\pi R^2}{2} - \sin^{-1} \left( \frac{a}{R} \right) + a^3 \log \left( \frac{R + \sqrt{[R^2 - a^2]}}{a} \right) - 2a\sqrt{[R^2 - a^2]} \right] \\ &= \frac{Rh}{3(R-r)} \left[ \frac{\pi R^2}{2} - R^2 \sin^{-1}(r/R) + \frac{Rr^3}{R-r} \log \left( \frac{R + \sqrt{[R^2 - r^2]}}{r} \right) - 2r\sqrt{[R^2 - r^2]} \right]\end{aligned}$$

## SOLUTIONS OF PROBLEMS.

### ARITHMETIC.

93. Proposed by **RAYMOND D. SMITH**, Tiffin, Ohio.

A barn 20 feet square is standing in a pasture, and a horse is tied to one corner of it with a rope 50 feet long. Over how much land can he graze?

I. Solution by **B. F. FINKEL**, M. Sc., M. A., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Let  $ABCD$  be the barn, side  $AB=AD=20$  feet;  $A$  the corner to which the horse is tied; and  $AF=AG=50$  feet, the length of the rope.

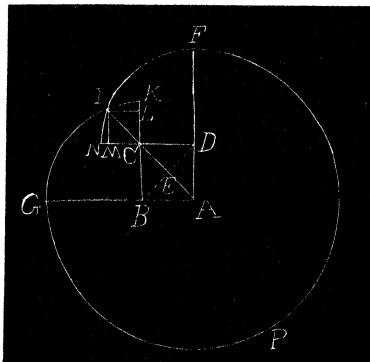
Then  $DI=BI=30$  feet;  $AC=DB=20\sqrt{2}$  feet;  $EI=\sqrt{[DI^2-DE^2]}=\sqrt{[30^2-(10\sqrt{2})^2]}$  feet  $=10\sqrt{7}$  feet;  $CI=EI-EC=10\sqrt{7}$  feet  $-10\sqrt{2}$  feet  $=10(\sqrt{7}-\sqrt{2})$  feet;  $CL=CM=\sqrt{[CI^2/2]}=\frac{1}{2}CI\sqrt{2}=5(\sqrt{14}-2)$  feet;  $KL=CK-CL=10$  feet  $-5(\sqrt{14}-2)$  feet  $=5(4-\sqrt{14})$  feet; and chord  $KI=\text{chord } IN=\sqrt{[KL^2+IL^2]}$ .

$\sqrt{[25(4-\sqrt{14})^2+25^2(\sqrt{14}-2)^2]}$  feet  $=10\sqrt{[3(4-\sqrt{14})]}$  feet.

$2 \text{ arc } IK = \frac{1}{3}(8 \text{ chord } KI - 2IL^*)$   
 $= \frac{1}{3}\{80\sqrt{[3(4-\sqrt{14})]} - 20(\sqrt{7}-\sqrt{2})\}$  feet  $= \frac{2}{3}\{4\sqrt{[3(4-\sqrt{14})]} - (\sqrt{7}-\sqrt{2})\}$  feet.

The area over which the horse can graze  $= FAGPF + \text{sector } FDI + \text{sector } IBG + \text{triangle } DCI + \text{triangle } BCI = FAGPF + 2 \text{ sector } FDI + 2 \text{ triangle } DCI = FAGPF + 2(\text{quadrant } FDN - \text{sector } IDN) + 2 \text{ triangle } DCI$ .

But area of  $FAGPF = \frac{3}{4}\pi AF^2 = 1875\pi$ ;



\*See *Williamson's Differential Calculus*, pages 84-85, for a proof of this rule. The discovery of this important approximation is due to Huygens. The length of an arc of  $30^\circ$  on a circle of radius 100,000 differs from the true value, assuming  $\pi=3.141592$ , by about 2 inches. The formula is  $\text{arc}=\frac{1}{3}(8B-A)$  when  $B$  is the chord of half the arc and  $A$  is chord of the arc.

area of quadrant  $FDN = \frac{1}{4}\pi DF^2 = 225\pi$ ; and area of sector  $IDN : 2\pi DF^2 :: 2\pi DF$  : arc  $IN$ , or area of sector  $IDN = \frac{1}{2}DF \times \text{arc } IN = 15\{ \frac{1}{3}[4\sqrt{3}[(4-\sqrt{14})] - \sqrt{7} + \sqrt{2}] \}$  square feet  $= 50\{4\sqrt{3}[3(4-\sqrt{14})] - \sqrt{7} + \sqrt{2}\}$  square feet.

$\therefore$  Area of sector  $FDI = 225\pi - 50\{4\sqrt{3}[3(4-\sqrt{14})] - \sqrt{7} + \sqrt{2}\}$ .

Area of triangle  $DCI = \frac{1}{2}DC \times IM = 10 \times 5(\sqrt{14}-2) = 50(\sqrt{14}-2)$  square feet.  $\therefore$  The total area over which the horse can graze  $= 1875\pi + 2(225\pi - 50\{4\sqrt{3}[3(4-\sqrt{14})] - \sqrt{7} + \sqrt{2}\}) + 2[50(\sqrt{14}-2)] = 1875\pi + 450\pi - 1004\sqrt{3}[3(4-\sqrt{14})] - \sqrt{7} + \sqrt{2} + 100(\sqrt{14}-2) = 2375\pi + 100\sqrt{14} - 2 - 4\sqrt{3}[3(4-\sqrt{14})] + \sqrt{7} - \sqrt{2} = 7249.378$  square feet.

II. Solution by G. I. HOPKINS, A. M., Professor of Mathematics and Physics, High School, Manchester, N. H.; J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.; and P. S. BERG, Principal of School, Larimore, N. D.

Let  $ABCD$  represent the barn and  $A$  the corner to which the horse is tied. Then  $FA = AG = 50$  feet, and  $DF = ID = BI = GB = 30$ . The area over which he can graze is divided into four parts, viz.: the three-quarters of a circle  $AFPGA$ , the two sectors  $GBI$  and  $IDF$ , and the quadrilateral  $IRCD$ .

$BD = \sqrt{(20^2 + 20^2)} = 20\sqrt{2}$ .  $\therefore ED = 10\sqrt{2}$ .

$\therefore CE = \sqrt{[30^2 - (10\sqrt{2})^2]} = 10\sqrt{7}$ .

Area  $IBCD = \text{area } IBD - \text{area } BCD$ .

$\therefore$  Area  $IBCD = 10\sqrt{7} \times 10\sqrt{2} - 200 = 100\sqrt{14} - 200$ .

$\therefore$  Area  $IBCD = 174.1657$  square feet.

$\cos \angle IDE = (10\sqrt{2})/30 = .4714$ .

$\therefore \angle IDE = 61^\circ 52' 30''$  and  $\angle BDA = 45^\circ$ .

$\therefore \angle IDA = 106^\circ 52' 30''$ .  $\therefore \angle IDF = 73^\circ 7' 30''$ .

Sectors  $GIC$  and  $IDF$  are equal.  $\therefore 2 \angle 73^\circ 7' 30'' = 146^\circ 15' = 146\frac{1}{4}^\circ$ .

Area of circle whose radius is  $ID = 30^2\pi = 900\pi$ .  $\therefore$  The areas of the two sectors  $GBI$  and  $IDF = (146\frac{1}{4}/360) \times 900\pi = 365\frac{1}{2}\pi = 365.625\pi$ .

Area of  $GAFIG = (3 \cdot 50^2\pi)/4 = 1875\pi$ .  $(365.625 + 1875)\pi = 2240.625\pi$ .

$\therefore$  Area  $GAFIG = 7039.1475$  square feet.

$\therefore$  Entire area  $= 174.1657 + 7039.1475 = 7213.3132$  square feet  $= 26.495$  square rods.

This problem was also solved by G. B. M. ZERR who got as an answer 7291.9868 square feet; J. Scheffer, his answer being 6889.414 square feet; Fremont Crane, his result being 6351.785 square feet; and B. F. Sine, his result being 7233.292 square feet. Cooper D. Schmitt did not solve it, but referred to a previous solution in the MONTHLY.

94. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics in Curry University, Pittsburg, Pa.

What rate of income do I realize by purchasing United States 4% bonds at 105 if I sell them in six years at 104?

Solution by CHARLES C. CROSS, Libertytown, Md.; FREMONT CRANE, Sand Coulee, Mont.; HON. JOSIAH H. DRUMMOND, Portland, Me.; and G. B. M. ZERR, Pottstown, Pa.

$.04 \times 6 = 24$ .

$1.04 + .24 = 1.28$ , amount realized on bond.

$1.28 - 1.05 = .23$ , amount gained in six years.